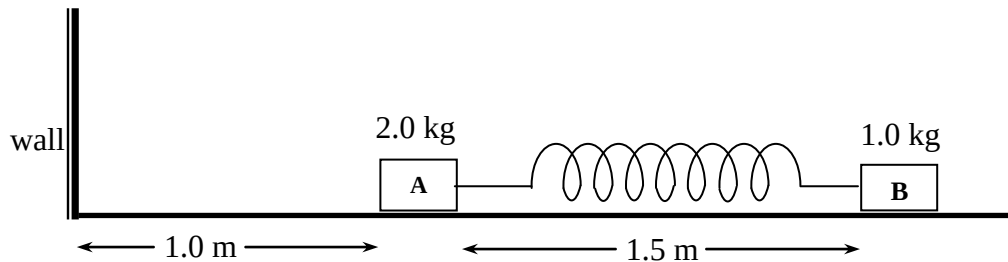


I. 1 (2 points)

Masses A and B attached to the ends of a stretched spring of negligible mass are released on a frictionless floor at the same instant as when A is 1.0 m from the wall and the spring is 1.5 m in length.

How far is A from the wall when the spring has contracted to 1.2 m in length?

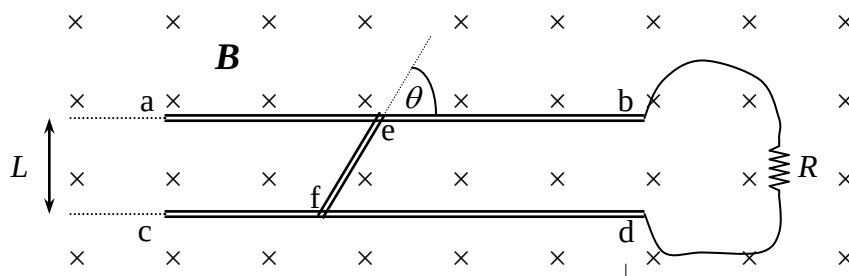


I. 2 (2 points)

ab and cd are smooth parallel conducting rods whose separation is L . The tips b and d are electrically connected through a resistance R . A uniform magnetic field $\mathbf{B}$ is directed inward perpendicularly to the plane abcd. Another conducting rod ef of mass M is constrained to slide without friction to the right maintaining a constant relative angular orientation θ and with initial velocity of $\mathbf{v}_0$ at time $t = 0$.

(a) Find the initial induced emf. (0.5 point)

(b) Find the velocity $\mathbf{v}$ at a later time t . (1.5 points)

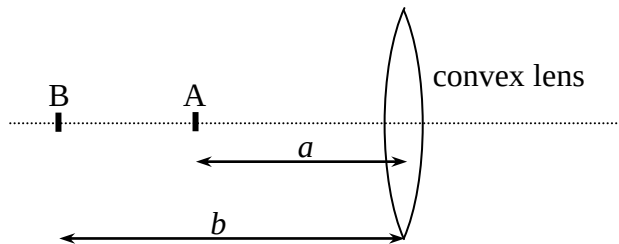


NOTE: $\int \frac{1}{x} dx = \ln|x| + C$

I.3 (2 points)

When an object is placed at A in front of a thin convex lens, the size of its virtual image is the same as that of the real image when the object is placed at B.

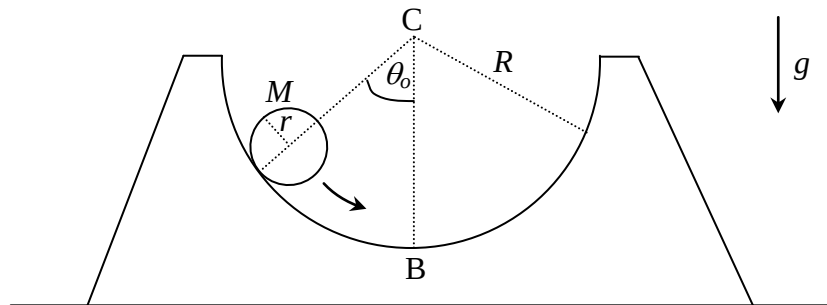
Find the expression for the focal length (f) of the convex lens in terms of a and b .



I.4 (2 points)

A solid sphere of mass M and radius r rolls without skidding from rest, when its angular position is θ_0 , down towards the bottom (B) of a spherical bowl of internal radius R .

Derive the expression for the maximum angular speed attained by M about the axis passing through its centre of mass, given that the moment of inertia of the solid sphere about an axis through its centre of mass is $\frac{2}{5}Mr^2$.

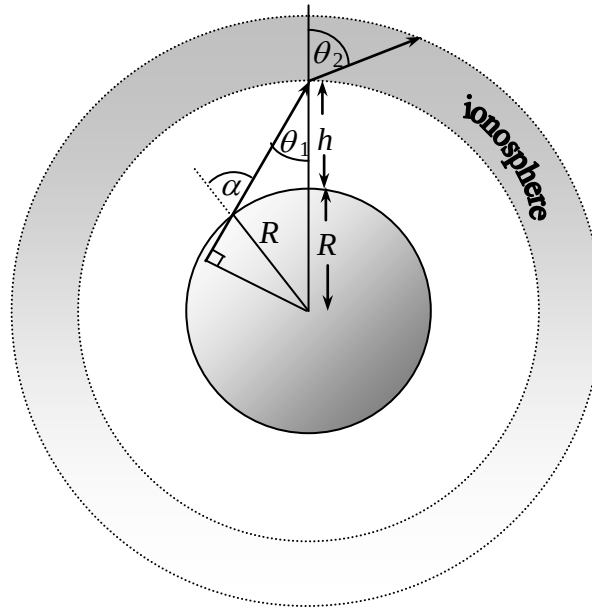


I. 5 (2 points)

The index of refraction of the ionosphere for the radio waves can be expressed as

$$n_{\text{ionosphere}} = \sqrt{1 - \frac{\lambda^2}{A}}$$

where A is a constant and λ is the wavelength of a wave. Assuming the ionosphere has a sharp lower boundary at an altitude h above the surface of the earth.



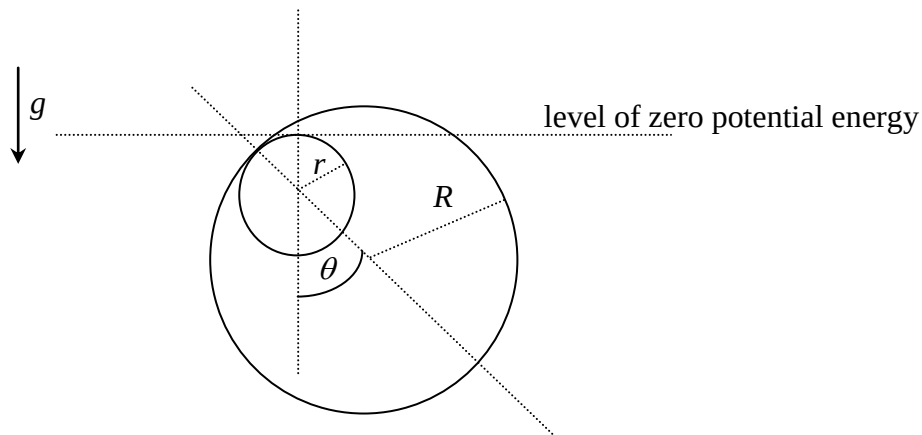
Consider a wave leaving the earth at an angle α with respect to the vertical, as shown in figure. Let θ_1 be an angle of incidence of this wave at the lower boundary of the ionosphere.

Find the shortest radio wavelength which can be totally reflected by the ionosphere when sent off from the surface of the earth at a given angle with respect to the vertical. This wavelength is called the *limiting wavelength*.

II. (10 points)

A cylindrical rod of radius r is fixed perpendicularly to a vertical wall. A thin circular hoop of mass M and radius R ($R > r$) can swing, in a vertical plane, about the rod without slipping.

Write down (with explanation of how to obtain) the expressions for the following quantities:



- the potential energy of the hoop (1 point)
- the kinetic energy of the hoop (2 points)
- the angular acceleration $\frac{d^2\theta}{dt^2}$ (4 points)

[Hint: either by differentiating the total mechanical energy of the system with respect to time or by using the torque equation about an appropriate point.]

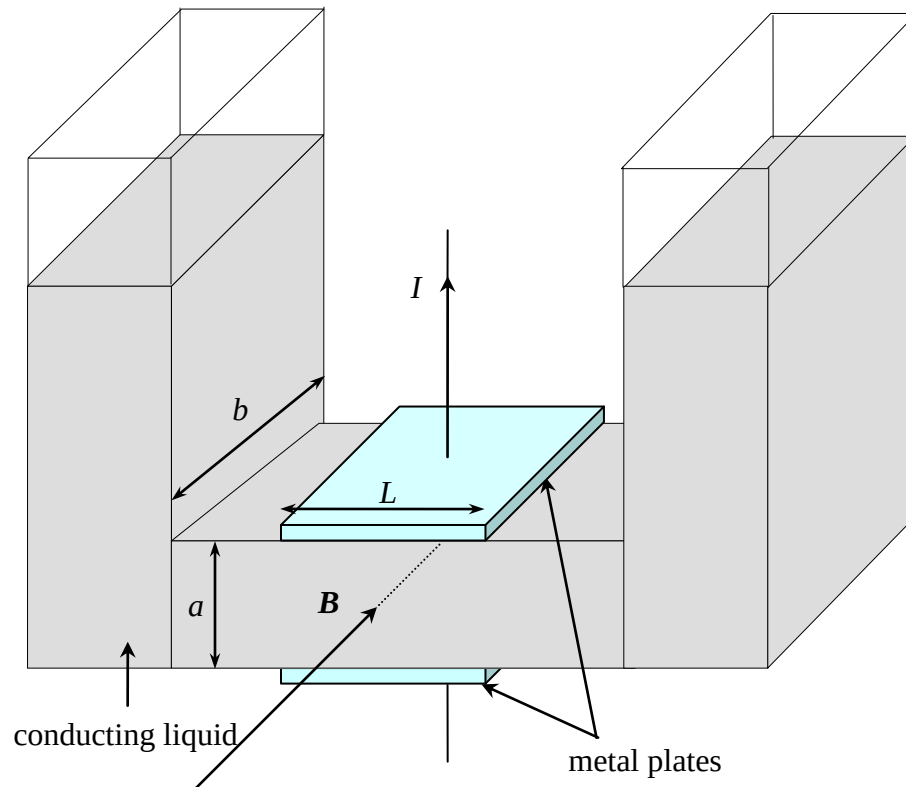
- the frequency of small-amplitude oscillation (2 points)
- deduce the frequency of oscillation for the case when the rod is reduced to a pin-sized rod. (1 point)

[Hint: $\frac{d^2x}{dt^2} = -k^2x$ has a solution of the form $x(t) = A\sin(kt + B)$]

[Given: Parallel axis theorem: $I = I_{cm} + mb^2$, where b is the distance between the new axis and the axis passing through the center of mass.]

III. (10 points)

The schematic diagram below shows a tube of rectangular cross-section of internal dimensions a and b . The tube is made of insulating material and contains liquid of density D and electrical conductivity σ . Over a length L of the tube, the top and bottom walls are made of metal, so that an electrical current, I , can be passed through the liquid. A uniform magnetic field of flux density B is applied in a direction perpendicular to and into the plane of the diagram over the region between the metal plates.



- In case of direct current, determine whether the liquid in the right-hand arm falls or rises. Explain how you obtained your answer. (3 points)
- Derive the expression for the difference in levels of the fluid in the two arms, h , in terms of I , B , D , b and g . (4 points)
- Obtain an expression for the heat dissipation in unit time in the liquid for a given value of h . In a particular case, $a = b = 1.0$ cm, $L = 10$ cm, $h = 1.0$ cm, $B = 1.0$ T, the liquid is an aqueous solution with $\sigma = 1.0 \Omega^{-1}\text{m}^{-1}$ and $D = 1.2 \times 10^3 \text{ kg/m}^3$. Calculate the amount of heat that could be dissipated in unit time in the liquid. Use $g = 9.8 \text{ m/s}^2$. (3 points)

NOTE: Conductivity is the reciprocal of resistivity
